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ANALYSIS OF THE MAXIMUM
PERFORMANCE OF A PARABOLOIDAL SOLAR
COLLECTION SYSTEM FOR SPACE POWER

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#### SUMMARY

An analytical study was performed to investigate the effects and interactions of the concentrator surface errors and rim angle, collection system orientation error, and cavity receiver operating temperature on the maximum thermal efficiency of a paraboloid collection system operating in the vicinity of the earth.

The ranges investigated were: standard deviation of surface error (0 to 18 min), orientation error (0 to 30 min), receiver temperature  $(2000^{\circ} \text{ to } 4000^{\circ} \text{ R or } 1110^{\circ} \text{ to } 2200^{\circ} \text{ K})$ , and concentrator rim angle (45 to 60 deg).

Results indicate that the surface error, orientation error, and receiver operating temperature each decidedly affect the collection efficiency and that these effects are interdependent. It is shown that surface and orientation error became increasingly important with increasing receiver operating temperature. A variation in rim angle, on the other hand, produces only a slight variation in collection efficiency and does not materially modify the effects of the other three parameters.

This information can be applied to the more comprehensive design optimization of a solar power system with regard to such factors as weight, size, and manufacturing simplicity.

#### INTRODUCTION

A reliable, long life, space power system capable of supplying sizable quantities of electrical power will be required to meet the needs of some more ambitious future space missions. One source of energy adequate for such a long duration space power system is the sun. Solar cell arrays are presently the only operational systems available for

converting solar energy into electric power.

However, various other power systems consisting of thermoelectric, thermionic, or turbodynamic devices could be employed to convert solar heat to electrical energy. These devices, which must operate at elevated temperatures, have been considered for systems with output power levels up to 40 kilowatts. The effective utilization of the available solar energy in the vicinity of the earth for high temperature power systems requires a concentration of this relatively low-intensity solar radiation.

A collection system must be employed to concentrate and supply the needed solar energy to the conversion system. The most widely applied collection system with the greatest potential for higher temperature operation consists of a paraboloid concentrating solar energy into a cavity receiver whose aperture is located in the paraboloid's focal plane. This system is theoretically capable of the highest concentration of solar energy and should result in the minimum receiver losses.

The collection system in any of the various power systems accounts for a substantial fraction of the total weight and volume. Maintaining the lowest possible weight and volume is a prime consideration for space application which encourages the use of an optimized collection system.

Several investigators have analyzed the performance of a paraboloidal solar collection system (refs. 1 to 3). They have omitted the effect of the subtended angle of the sun and/or assumed that the solar radiation is reflected on the focal plane with a normal distribution without relating this to any specific surface accuracy or physical condition of the concentrator. Each of these factors affects the quantity of energy absorbed by the receiver.

The determination of collection efficiency in this analysis includes the effect of the subtended angle of the sun and directly relates the energy going into the receiver with the errors in the surface of the concentrator. A normal, or Gaussian, distribution of surface errors was assumed.

The effects of concentrator surface error and rim angle, orientation error, and receiver temperature on the performance of the collection system are investigated.

Individual and collective effects of these variables on the maximum efficiency of a collection system operating in the vicinity of the earth are analyzed, and the interactions between these variables are demonstrated.

The maximum collection efficiency was obtained through an exchange between captured and emitted radiation from the receiver as its aperture size was varied until the minimum collection system loss was achieved.

A concentrator surface accuracy ranging from excellent to poor (0 to 18 min standard deviation of surface error) and an orientation requirement ranging from strict to lenient (0 to 30 min) were considered in the evaluation. The operating temperature level was sufficient to apply to systems ranging from the relatively low temperature dynamic

to the high temperature thermionic systems (2000° to 4000° R or 1110° to 2220° K). The range of rim angles are considered to be compatible with low weight, high strength, and compact packaging (45 to 60 deg).

The results of this study describe the variation of optimum collection efficiency with each of the effective parameters thus establishing the minimum requirements for at - taining a prescribed performance.

This information can be utilized in the more comprehensive design optimization of a solar power system in regard to such factors as weight, size, and manufacturing simplicity.

# SYMBOLS

A	receiver aperture area, ft <sup>2</sup> (m <sup>2</sup> )
D	concentrator diameter, ft (m)
K	solar constant, 442 Btu/(hr)(ft $^2$ ); 1390 J/(m $^2$ )(sec)
Т	receiver operating temperature, <sup>O</sup> R ( <sup>O</sup> K)
$t_{o}/t_{s}$	ratio of total time that receiver operates with open aperture to sun time
$^{lpha}_{ extsf{s}}$	effective solar absorptivity of receiver
β	orientation error, min
$\epsilon_{ m T}$	effective thermal emissivity of receiver
${}^{\eta}{}_{\mathrm{B}}$	concentrator blockage factor
$\eta_{c-a}$	collection efficiency
$\eta_{ { m E}}$	fraction of energy reflected from concentrator entering receiver
${}^{\eta}{}_{ m R}$	reflectivity of concentrator
$^{ heta}\mathbf{R}$	concentrator rim angle, deg (see fig. 1)
σ	Stefan-Boltzmann constant, $1712\times10^{-12}$ Btu/(hr)(ft <sup>2</sup> )( $^{O}R^{4}$ ); $5.67\times10^{-8}$ J/(m <sup>2</sup> ) ( $^{O}K^{4}$ )(sec)
$^{\sigma}\mathbf{s}$	standard deviation of surface error, min

# COLLECTION SYSTEM DESCRIPTION

The configuration of the solar collection system is shown in figure 1. It consists of a paraboloid concentrating solar radiation into a cavity receiver whose aperture is posi-

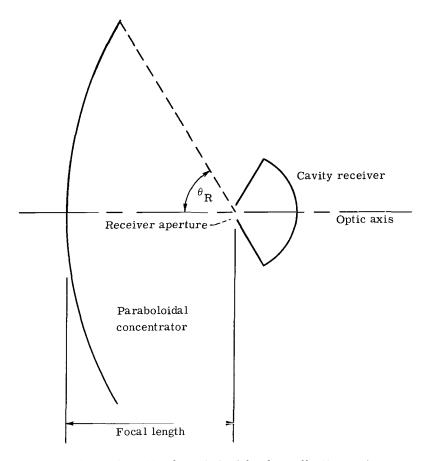


Figure 1. - Schematic of paraboloidal solar collection system.

tioned in the focal plane and centered along the optic axis of the concentrator.

In the vicinity of the earth, solar radiation is incident to each point of the collector over a cone angle of 32 minutes. Because of this property, the energy reflected from any single point of a perfect concentrator is spread over a small area of the focal plane. The total energy delivered to the focal plane is distributed with varying intensity.

Any concentrator surface error which exists will additionally diffuse the solar radiation over a larger area of the focal plane. Orientation error will mainly relocate the energy on the focal plane while also producing a slight diffusion.

Because of the high concentration of energy near the center of the reflected image, the amount of energy entering the receiver increases rapidly at first as the aperture is enlarged and then gradually until all the energy enters the receiver.

The solar receiver positioned with its aperture in the focal plane will gain energy by absorption of concentrated solar radiation and lose energy by emitting radiation.

The quantity of radiation absorbed by the receiver will vary with any change in the concentrator surface errors, rim angle, and the collection system orientation error.

The amount of energy emitted from the receiver will vary with a change in the operating temperature of the receiver.

The useful energy, that which is available to an energy conversion system, is the net between the absorbed and emitted radiation. The useful energy expressed as a fraction of the incident energy is given in terms of the collection efficiency which is calculated from the following:

$$\eta_{c-a} = \eta_B \eta_R \eta_E \alpha_s - \frac{\epsilon_T \sigma A}{\frac{\pi D^2}{A}} \frac{T^4}{K} \frac{t_o}{t_s}$$
 (1)

In this expression,  $\eta_B \eta_R \eta_E \alpha_s$  represents the fraction of energy incident to the collection system which is absorbed by the receiver and

$$\frac{\epsilon_{\mathrm{T}}^{\sigma}_{\mathrm{A}}}{\frac{\pi \mathrm{D}^{2}}{4}} \frac{\mathrm{T}^{4}}{\mathrm{K}} \frac{\mathrm{t}_{\mathrm{o}}}{\mathrm{t}_{\mathrm{s}}}$$

accounts for the fraction of incident energy emitted from the receiver by radiation. It is assumed that the receiver is insulated to reduce any other thermal losses to zero.

# ANALYSIS AND PROCEDURE

The predominant part of this analytical study is concerned with determining the maximum efficiency of a paraboloidal collector concentrating solar energy into a blackbody cavity receiver as a function of (1) the paraboloidal concentrator rim angle, (2) surface errors, (3) collection system orientation error, and (4) receiver operating temperature. A brief investigation of the influence of the effective emissivity of the receiver is also included.

A cavity receiver has the characteristic of minimizing reflection and radiation losses from the receiver. Cavity receiver characteristics were analyzed (ref. 4), and it was determined that the ideal blackbody behavior can be very closely approximated with real avities of reasonable size. Therefore, the present analysis has assumed that  $\epsilon_T = 1$  and  $\alpha_S = 1$ .

The concentrator surface errors are assumed to follow a normal distribution. Experience with concentrator fabrication and inspection (1ef. 5) have shown that this assumption of a normal distribution of surface error is a very reasonable approximation of real concentrators.

The collector for this analysis is considered to be unobstructed ( $\eta_{\rm B}$  = 1.0). The concentrator reflectivity is assumed to be 0.9, a value which can be expected from presently available coatings.

The collection efficiency was calculated for a system operating in the vicinity of the earth with its total period of operation in the sun. Equation (1) also applies to a system operating in an orbit with a shade cycle if the receiver aperture is maintained shut during the dark portion of the orbit (i. e.,  $t_{\rm c}/t_{\rm s}=1$ ).

The iraction of available energy entering the receiver  $\eta_E$  varies with the size of the receiver aperture, the concentrator surface error and rim angle, and the system orientation error. The quantity  $\eta_E$  is calculated by utilizing a method of analysis based on cone optics as reported in reference 6.

An example of the variation of  $\eta_{\rm E}$  with receiver aperture size is illustrated in figure 2 for a specific set of conditions. Because of the high concentration of energy near the center of the reflected image, the amount of energy entering the receiver increases rapidly at first as the aperture is enlarged and then gradually until all the energy enters the receiver. The pattern demonstrated in this example is similar for other values of concentrator surface error and rim angle and system orientation error.

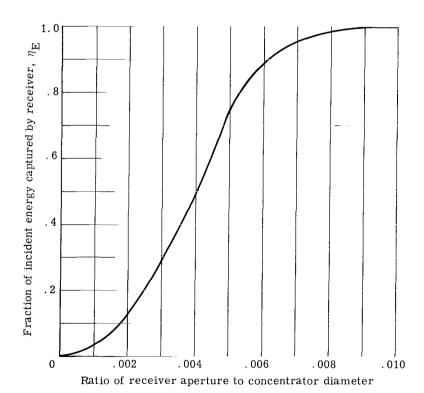


Figure 2. - Quantity of energy into receiver with varying aperture. Concentrator rim angle, 55 degrees; orientation error, 0; standard deviation of surface error, 0.

The diameter of the receiver aperture strongly affects the efficiency of the collection system. While the fraction of available energy entering the receiver increases by enlarging the receiver aperture (see fig. 2), the radiation emitted from the receiver is simultaneously increasing.

A computer program was utilized to perform an exchange between absorbed and emitted radiation by varying the size of the receiver aperture and determining the net useful energy according to equation (1). The optimum exchange between the two quantities results in the maximum collection efficiency. An example of the manner by which the performance of the collection system varies with receiver aperture size is illustrated in figure 3 for three receiver operating temperatures.

Using the described procedure, the collection efficiency as a function of receiver aperture size was determined varying the paraboloidal collector rim angle over the range

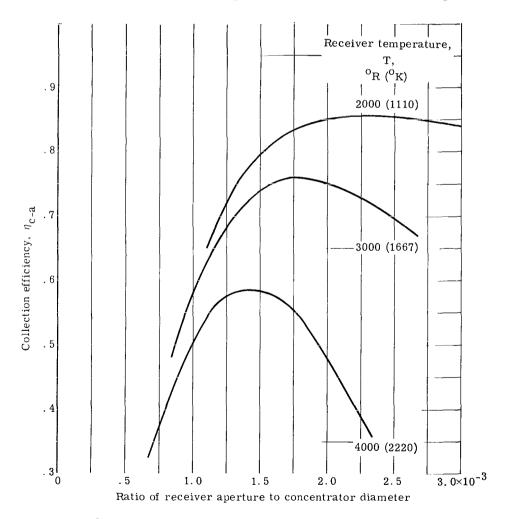


Figure 3. - Effect of aperture size on collector efficiency. Concentrator rim angle, 50 degrees; orientation error, 15 minutes; standard deviation of surface error, 6 minutes.

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of 45 to 60 degrees, the collector standard deviation of surface error over the range of 0 to 18 minutes, the collection system orientation error over the range of 0 to 30 minutes, and the receiver operating temperature over the range of  $2000^{\circ}$  to  $4000^{\circ}$  R (1110 to  $2220^{\circ}$  K). Numerous curves similar to figure 3 were obtained for each combination of the mentioned variables and then cross plotted to obtain the optimum variation of collection efficiency shown in the accompanying graphs.

# RESULTS AND DISCUSSION

The effects of collector surface error, orientation error, rim angle, and receiver operating temperature on the performance of the collection system were studied and the variation in optimum efficiency with these parameters is shown in figure 4. These are the basic results of this analysis. Also shown in figure 4 are the maximum attainable efficiencies in the absence of surface and orientation errors. These values serve as a standard of comparison to demonstrate the limits of performance and the degradation in maximum performance that result when errors are introduced into the system.

# Surface Error

The peak efficiency decreases steadily and appreciably with increasing surface error. As shown in figure 4(d), an increase in the surface error from 6 to 12 minutes at a temperature of  $2000^{\circ}$  R ( $1110^{\circ}$  K) and an orientation error of 15 minutes reduces the efficiency from 0.85 to 0.81. Increasing the magnitude of the concentrator surface errors increases the dispersion of solar radiation reflected from the concentrator such that a reduced quantity of energy is absorbed by the receiver (for any given aperture size), correspondingly decreasing the collection efficiency.

#### Orientation Error

The peak efficiency decreases appreciably and steadily with increasing orientation error. As shown in figure 4(d), an increase in the orientation error from 15 to 30 minutes at a temperature of  $2000^{\circ}$  R ( $1110^{\circ}$  K) and a surface accuracy of 6 minutes results in a drop of efficiency from 0.85 to 0.825. An increase in the orientation error mainly relocates the energy delivered to the focal plane and also slightly diffuses the solar radiation reflected from the concentrator so that a smaller quantity of energy is absorbed by the receiver (of a given aperture size) thereby resulting in a lower collection efficiency.

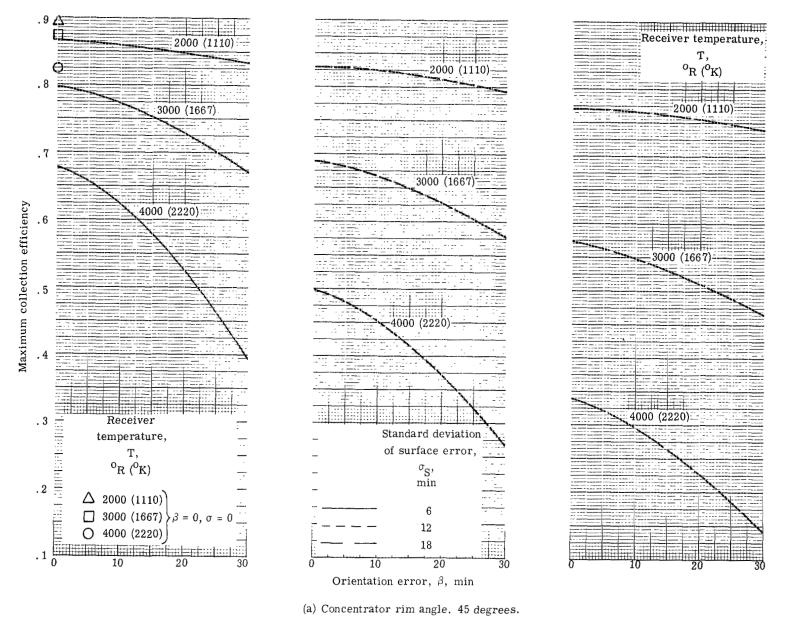
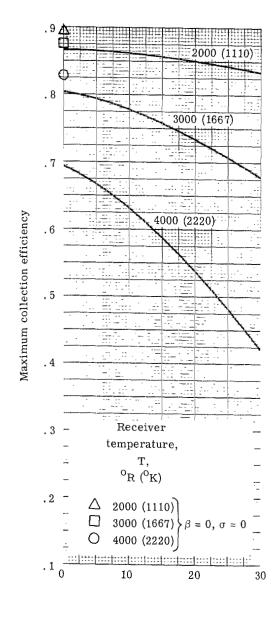
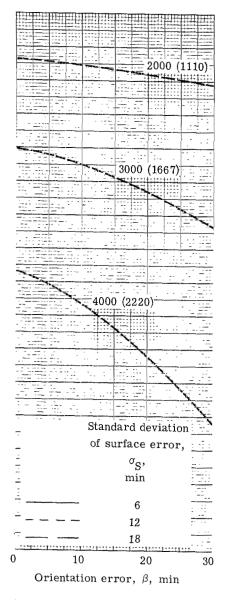
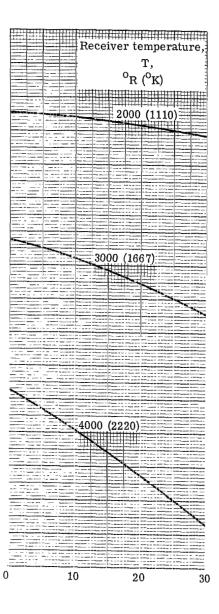


Figure 4. - Variation of maximum collection efficiency of paraboloidal collection system with orientation error, surface error, and temperature.

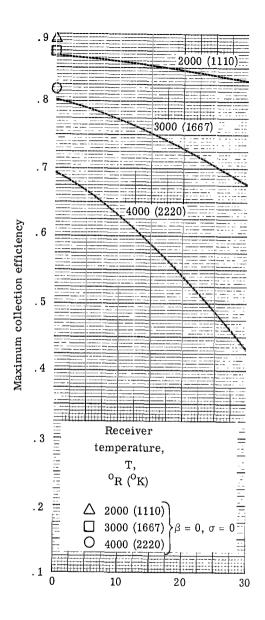


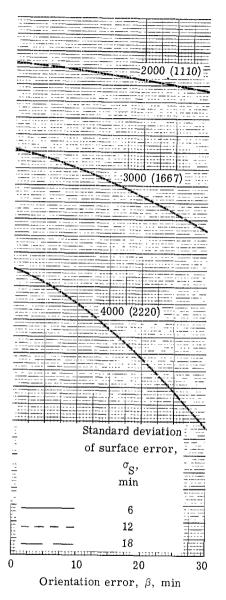


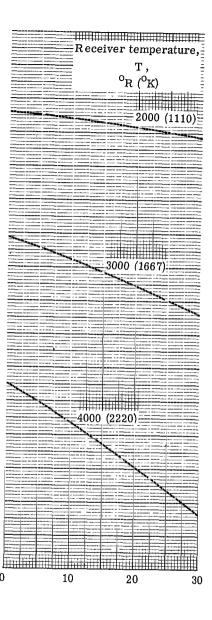


(b) Concentrator rim angle, 50 degrees.

Figure 4. - Continued.

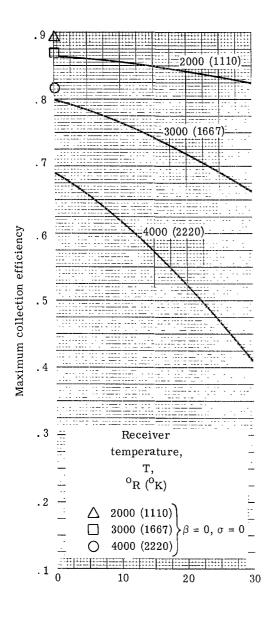


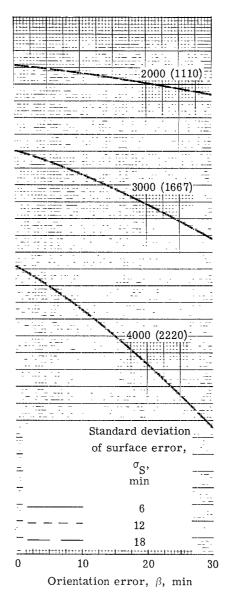


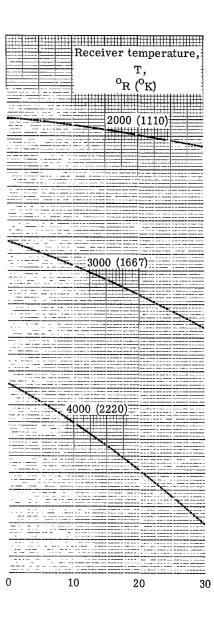


(c) Concentrator rim angle, 55 degrees.

Figure 4. - Continued.







(d) Concentrator rim angle, 60 degrees.

Figure 4. - Concluded.

# Effect of Combined Surface and Orientation Error

It is observed that the deterioration in collection efficiency for an identical increase of orientation error is larger at higher values of concentrator surface error. It can be observed in figure 4(d) at a temperature of  $4000^{\circ}$  R ( $2220^{\circ}$  K) and a surface error of 6 minutes that the collection efficiency drops from 0.69 to 0.41 for a difference of 0.28 or 40 percent when the orientation error increases from 0 to 30 minutes. At the same temperature and a surface error of 18 minutes the same increase in orientation error reduces the collection efficiency from 0.38 to 0.17 for a difference of 0.21 or 55 percent.

# Effect of Temperature

The maximum collection efficiency decreases with increasing receiver operating temperature. The radiation losses are directly proportional to the fourth power of tem-

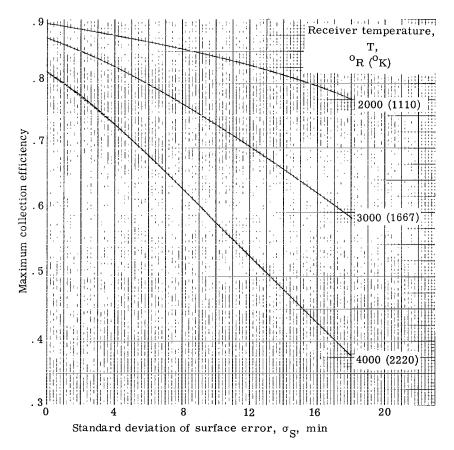


Figure 5. - Effect of collector surface error on maximum collection efficiency. Collector rim angle, 60 degrees; orientation error, 0.

perature as shown by equation (1).

As shown in figure 4, an increase in temperature at higher levels of orientation error results in larger losses in collection efficiency. For example, it can be seen from figure 4(d) that when the surface error is 6 minutes and the orientation error is 15 minutes, an increase in temperature from  $2000^{\circ}$  to  $4000^{\circ}$  R ( $1110^{\circ}$  to  $2220^{\circ}$  K) results in a drop in efficiency from 0.85 to 0.575 for a difference of 0.275 or 32 percent. However, with an orientation error of 30 minutes, the efficiency decreases from 0.825 to 0.41 for a difference of 0.415 or 50 percent as the temperature increases from  $2000^{\circ}$  to  $4000^{\circ}$  R ( $1110^{\circ}$  to  $2220^{\circ}$  K).

Similarly, it can be observed in figure 5, that the deterioration in maximum performance with increasing receiver operating temperature is intensified at larger surface errors. At zero orientation error and 6-minute standard deviation of surface error, the efficiency drops from 0.865 to 0.69 for a change of 0.175 or 20 percent when the temperature increases from 2000° to 4000° R (1110° to 2220° K). For a standard deviation of surface error of 12 minutes the efficiency drops from 0.827 to 0.530 for a change of 0.297 or 36 percent when the temperature increases from 2000° to 4000° R (1110° to 2220° K).

Larger surface and orientation errors result in larger receiver apertures which multiply the radiation losses from the receiver and thereby decrease the collection efficiency by increasing quantities for any given increase in temperature.

# Effect of Rim Angle

Figure 6 shows the variation in maximum collection efficiency with rim angle for four combinations of surface and orientation errors at three receiver operating temperatures. In general, there is slight variation in maximum collection efficiency with rim angle. At low values of surface error, orientation error, and temperature, the maximum collection efficiency is practically insensitive to a change in rim angle. As the errors and temperature increase a variation in maximum collection efficiency with a change in rim angle becomes noticeable with the peak value of efficiency shifting towards the higher rim angle. For a surface accuracy of 6 minutes and an orientation error of 15 minutes at an operating temperature of  $3000^{\circ}$  R ( $1667^{\circ}$  K) the variation in maximum efficiency with a change in rim angle from 45 to 60 degrees is just slightly over one percentage point. The peak efficiency occurs at a rim angle of about 50 degrees. The largest variation in collection efficiency shown in figure 6 is less than 0.03, and this occurs at levels of efficiency beyond the range of practical interest.

Increasing the rim angle of the concentrator redistributes the reflected energy increasing the intensity towards the center of the reflected image while at the same time

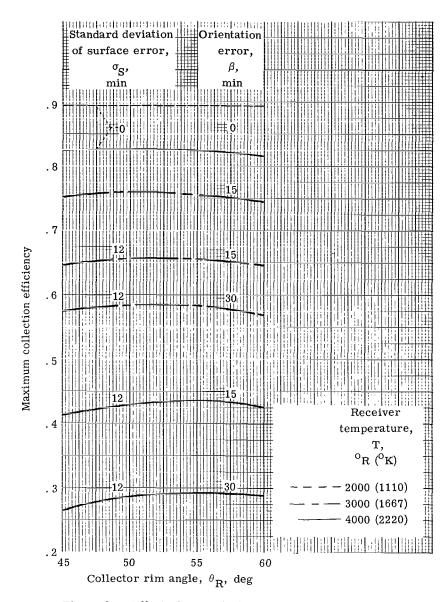


Figure 6. - Effect of concentrator rim angle on maximum collection efficiency.

extending the area of the focal plane to which energy is delivered. The effect is that small receiver apertures will capture more energy with the higher rim angle concentrators. As the receiver apertures increase, the reverse effect is prevalent. This results in only a slight change in the amount of energy going into the receiver at its optimum aperture, and accordingly, the maximum collection efficiency varies slightly.

The effect of the rim angle on collection efficiency is minor and the choice of rim angle may be made on other factors.

# Influence of Effective Emissivity

The losses from a receiver consist of emitted radiation and directly reflected radiation. In this study the cumulative effect of these two losses from the receiver are accounted for through a single convenient quantity defined as the effective emissivity. This is the apparent emissivity of the receiver aperture which will result in the total losses.

A cavity receiver may be constructed so that for its aperture its effective emissivity is essentially unity. It is not, however, evident how a lower effective emissivity can be obtained except possibly through the use of a material with selective radiation properties in a specific receiver configuration.

It is however readily noticeable by examination of equation (1) that a reduction in effective emissivity would contribute directly to reducing the losses from the receiver.

One example of the degree of improvement in collection efficiency obtained by reducing the effective emissivity of the receiver is shown in figure 7. The maximum collection efficiency is plotted as a function of orientation error for various values of effective emissivity, at a temperature of 2000° R (1110° K), a surface error of 6 minutes and a rim angle of 60 degrees. It can be observed that at the low values of orientation error,

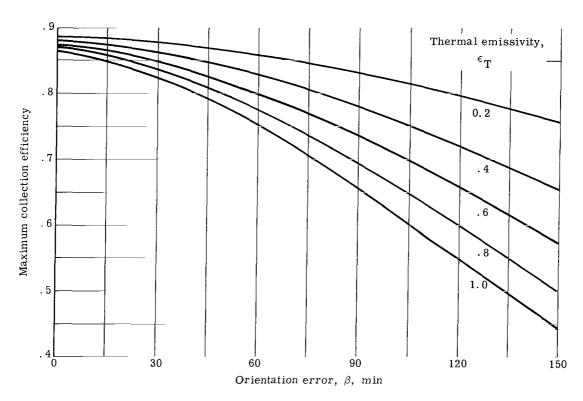


Figure 7. - Influence of effective emissivity on maximum collection efficiency. Concentrator rim angle, 60 degrees; standard deviation of surface error, 6 minutes; receiver temperature,  $2000^{\circ}$  R ( $1110^{\circ}$  K).

a minor improvement in collection efficiency occurs with a reduction in effective emissivity. The improvement in efficiency grows significantly as the orientation error increases.

The gain in efficiency with lower effective emissivity may be used to extend the tolerable range of orientation error necessary to maintain a given level of performance. For instance, figure 7 discloses that by reducing the effective emissivity from 1.0 to 0.2 the allowable orientation error may pass from 30 to 95 minutes and the same efficiency of 0.825 will be obtained.

Similarly, the reduction in effective emissivity may be used to cut down the required level of concentrator accuracy and or increase the operating temperature in attaining a prescribed level of collection efficiency.

#### CONCLUSIONS

A parametric analysis was performed of the effects of concentrator surface errors and rim angle, collection system, orientation error, and receiver operating temperature on the maximum efficiency of a paraboloidal collection system operating in the vicinity of the earth. The following conclusions were reached:

- 1. The effect of surface error and orientation error are intimately connected with the receiver operating temperature and grow in importance as the temperature increases.
- 2. The effect of concentrator rim angle on efficiency is slight and is essentially independent of the other parameters. Hence, concentrator rim angle may be chosen on the basis of weight, strength requirement, or manufacturing simplicity.
- 3. As the operating temperature of the system increases, a higher orientation and surface accuracy will be required to obtain a desired level of efficiency.
- 4. For identical increases in orientation error, larger percentage reductions in collection efficiency occur with an increasingly inaccurate concentrator.
- 5. If the effective emissivity of the receiver could be reduced, the level of orientation error or surface error can be increased without penalty to collector-receiver efficiency. Similarly the operating temperature may be increased without a penalty to the collection efficiency.

Lewis Research Center,

National Aeronautics and Space Administration, Cleveland, Ohio, October 25, 1967, 120-33-05-02-22.

# REFERENCES

- 1. Silvern, David H.: An Analysis of Mirror Accuracy Requirements for Solar Power Plants. Paper No. 1179-60, ARS, May 1960.
- 2. McClelland, D. H.; and Stephens, C. W.: Solar-Thermal Energy Sources. Vol. 2 of Energy Conversion Systems Reference Handbook. Rep. no. 390, vol. 2 (WADD TR 60-699, vol. 2, DDC no. AD-256973), Electro-Optical Systems, Inc., Sept. 1960.
- 3. Liu, B. Y. H.; and Jordan, R. C.: Performance and Evaluation of Concentrating Solar Collectors for Power Generation. J. Eng. Power, vol. 87, no. 1, Jan. 1965, pp. 1-7.
- 4. Stephens, Charles W.; and Haire, Alan M.: Internal Design Consideration for Cavity-Type Solar Absorbers. ARS J., vol. 31, no. 7, July 1961, pp. 896-901.
- 5. Kovalcik, E. S.: Brayton Cycle Solar Collector Design Study. Rep. No. ER-5938 (NASA CR-54118), Thompson Ramo Wooldridge, Inc., Mar. 1964, pp. 14-15.
- 6. Kaykaty, Gabriel N.: General Method for Predicting Efficiency of Paraboloidal Solar Collector. NASA TM X-1323, 1966.

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